



Schrödinger-Type Transmission Line Equation via Physics-Informed Neural Networks (PINNs)

Persamaan Garis Transmisi Tipe Schrödinger melalui Jaringan Saraf Terinspirasi Fisika (PINNs)

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Abstract. Accurate modeling of high-frequency transmission systems requires efficient methods to represent dispersive and nonlinear effects. Traditional solvers like the Split-Step Fourier Method (SSFM) are accurate but computationally demanding. The Schrödinger-Type Transmission Line Equation (STLE) effectively models these systems, yet existing techniques struggle with inverse problems and limited data. Research on data-efficient and stable frameworks for STLEs remains limited. This study introduces a Physics-Informed Neural Network (PINN) to solve forward and inverse STLE problems by embedding physical laws into the training process. The model achieved a relative L^2 error $\leq 1 \times 10^{-3}$ and identified dispersion (β_2) and nonlinearity (γ) coefficients with 1–3% error using $\leq 5\%$ data. Gradient-regularized losses and Fourier-enhanced inputs improve training stability and precision. The proposed approach enables reliable, data-efficient modeling of nonlinear transmission systems for advanced electromagnetic design.

Keywords : Physics-Informed Neural Network, Schrödinger-type, split-step Fourier method, partial differential equation, transmission

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Abstrak. Modeling sistem transmisi frekuensi tinggi yang akurat memerlukan metode efisien untuk mewakili efek dispersif dan nonlinier. Solver tradisional seperti Split-Step Fourier Method (SSFM) akurat namun memerlukan komputasi yang intensif. Schrödinger-Type Transmission Line Equation (STLE) secara efektif memodelkan sistem ini, namun teknik yang ada kesulitan dalam menyelesaikan masalah invers dan data yang terbatas. Penelitian tentang kerangka kerja yang efisien dan stabil untuk STLE masih terbatas. Studi ini memperkenalkan Jaringan Saraf Terinspirasi Fisika (PINN) untuk menyelesaikan masalah maju dan invers STLE dengan mengintegrasikan hukum fisika ke dalam proses pelatihan. Model ini mencapai kesalahan L^2 relatif $\leq 1 \times 10^{-3}$ dan mengidentifikasi koefisien dispersi (β_2) dan nonlinieritas (γ) dengan kesalahan 1–3% menggunakan $\leq 5\%$ data. Kerugian yang diregulasi gradien dan masukan yang ditingkatkan Fourier meningkatkan stabilitas dan presisi pelatihan. Pendekatan yang diusulkan memungkinkan pemodelan yang andal dan efisien data untuk sistem transmisi nonlinier dalam desain elektromagnetik canggih.

Kata Kunci : Jaringan Saraf Terinspirasi Fisika, tipe Schrödinger, metode Fourier langkah terpisah, persamaan diferensial parsial, transmisi

1. Introduction

The rapid development of high-frequency transmission systems, encompassing optical fiber links and high-speed microwave and millimeter-wave interconnects, has intensified the demand for precise and computationally efficient models of dispersive and nonlinear propagation phenomena. The nonlinear Schrödinger-type transmission line equation (STLE) precisely characterizes the behavior of electromagnetic signals in these media, demonstrating the evolution of the complex field envelope influenced by group-velocity dispersion (GVD) and self-phase modulation (SPM). For many years, traditional numerical solvers such as the finite-difference time-domain (FDTD) method [1], the finite-element method (FEM) [2], and the split-step Fourier method (SSFM) [3] have provided valuable insights.. But these methods often have high computational costs, depend on the mesh, and are hard to use for inverse or parameter-identification problems when measurements are few.

The Schrödinger-type model is used in a lot of transmission applications. In optical fibers, it controls pulse broadening, soliton formation, and four-wave mixing [4]–[6]; in microwave integrated circuits, it models nonlinear dispersive wave propagation [7]; and in plasmonic and photonic crystal lines, it captures complex envelope dynamics under anomalous dispersion regimes [8], [9]. There are only a few cases where analytical solutions work (for example, fundamental solitons). This is why high-accuracy numerical schemes like adaptive step-size SSFM [10], high-order Runge–Kutta methods [11], and pseudospectral algorithms [12] were created. Even though these methods are well-established, they usually need finely discretized meshes and have trouble with inverse design or finding parameters that are different from each other.

Physics-Informed Neural Networks (PINNs), introduced by Raissi et al. [13], have emerged as a promising method for solving PDEs by directly integrating the governing physics into the learning objective. PINNs can learn the continuous solution space without having to break it up into pieces by punishing the PDE's residual, the boundary conditions, and the initial conditions. PINNs have been useful for a lot of problems since they were first used, like fluid dynamics [14], electromagnetics [15], quantum mechanics [16], and nonlinear optics [17]. These networks have proven adept at generalization, noise resilience, and managing incomplete data and irregular domains..

Many studies have tried to make PINNs more useful by adding wave propagation and Schrödinger-type equations. Chen et al. [18] employed PINNs on the 1D nonlinear Schrödinger equation, achieving accurate reconstruction of soliton dynamics. Lu et al. [19] presented DeepXDE, a proficient PINN framework developed to handle complex boundary conditions for wave equations. Yang and Karniadakis [20] introduced variational PINNs (VPINNs), which alleviate the inflexibility of high-order PDE learning by minimizing the weak form residual. Peng et al. [21] and Mao et al. [22] demonstrated that PINNs can ascertain unknown coefficients in dispersive systems, such as dispersion length and nonlinear index, solely by analyzing noisy measurements. These advancements present innovative opportunities for the integration of machine learning with electromagnetic transmission modeling..

Nonetheless, challenges persist in employing PINNs for high-frequency dispersive issues. Solutions of the Schrödinger type have an oscillatory nature that leads to gradient pathologies and phase errors in standard networks [23]. Long-distance propagation also causes cumulative numerical dispersion, which can make training less stable [24]. To

address these challenges, researchers have created spectral PINNs that use Fourier features [25], SIREN-based networks that use sinusoidal activation functions [26], and adaptive loss weighting or gradient regularization methods [27] to balance the learning of multi-scale physical terms. Recent hybrid formulations, such as Fourier neural operators (FNOs) [28], neural spectral solvers [29], and quantum-inspired PINNs [30], demonstrate improved efficacy in accelerating PDE learning for wave and transmission systems..

This paper introduces a Physics-Informed Neural Network framework for solving the Schrödinger-Type Transmission Line Equation (STLE). The proposed method establishes both forward and inverse problems, enabling precise reconstruction of the field and determination of the coefficients (dispersion and nonlinearity) in dispersive transmission environments. The network uses gradient-regularized loss to keep training stable when there are high-frequency oscillations and Fourier-enhanced inputs to quickly capture spectral dispersion. Comparative analyses utilizing the split-step Fourier method (SSFM) validate the accuracy and stability of the proposed approach. The rest of the paper is organized like this:

Section 2 talks about the STLE's governing equations and the physical background. The proposed PINN formulation, which includes the loss design and training plan, is shown in Section 3. In Section 4, there are numerical experiments and comparisons. Section 5 talks about what can be added, what can't be added, and what the future holds..

2. Governing Equation and Transmission Context

A PDE of Schrödinger-type partial differential equation can exactly characterize the propagation of high-frequency electromagnetic or voltage signals through dispersive and nonlinear broadcasting media, such as optical fibers,

coaxial cables, and microwave transmission lines. This formulation is based on equations of Maxwell's and slowly changing envelope estimate (SVEA), which declares that the envelope of the signal changes slowly in relation to its carrier frequency [26]–[28]. Let $\psi(x, t)$ symbolize the complex envelope of the electric field, voltage, or current along the transmission axis x at time t . The equation of Schrödinger-type transmission-line is inscribed similar this:

$$j \partial \psi / \partial t + (1/2) \beta_2 \partial^2 \psi / \partial x^2 - \gamma |\psi|^2 \psi = 0 \quad (1)$$

where $j = \sqrt{-1}$ is the imaginary unit, β_2 is the group-velocity-dispersion (GVD) coefficient, and γ is the nonlinearity coefficient that comes from self-phase modulation. The parameter β_2 describes how the propagation constant changes with frequency and controls how signals spread out over time in optical or electrical media [29], [30]. The coefficient γ takes into account changes in the refractive index of optical fibers that depend on intensity or changes in inductance and capacitance of electrical transmission lines that depend on voltage. Equation (1) follows directly from Maxwell's curl relations:

$$\nabla \times \mathbf{E} = -\mu \partial \mathbf{H} / \partial t, \quad \nabla \times \mathbf{H} = \epsilon \partial \mathbf{E} / \partial t + \mathbf{J} \quad (2)$$

Applying the **quasi-TEM** approximation for guided modes yields the *telegrapher's equations* for voltage $V(x, t)$ and current $I(x, t)$:

$$\begin{aligned} \partial V / \partial x &= -L \partial I / \partial t - R I, & \partial I / \partial x &= -C \partial V / \partial t - G V \end{aligned} \quad (3)$$

where L , C , R , and G are the per-unit-length inductance, capacitance, resistance, and conductance. Neglecting losses ($R, G \approx 0$) gives the familiar wave equation:

$$\partial^2 V / \partial x^2 = L C \partial^2 V / \partial t^2 \quad (4)$$

Introducing a carrier-wave representation $V(x, t) = \psi(x, t) \exp[j(\omega_0 t - \beta_0 x)]$ and keeping higher-order terms under the SVEA gives us the dispersive–nonlinear form (1). This link shows that when dispersion and nonlinearity are both present, both optical-fiber and electrical-line propagation follow the same Schrödinger-type dynamics.

The first term, $j \partial\psi/\partial t$, describes how the envelope changes over time. The second term, $(1/2) \beta_2 \partial^2\psi/\partial x^2$, controls group-velocity dispersion, which makes the pulse spread or compress depending on the sign of β_2 . The third term, $-\gamma |\psi|^2 \psi$, represents self-phase modulation, which causes phase shifts and waveform distortion that depend on the intensity. When $\gamma = 0$, Equation (1) becomes a linear dispersive model. When $\gamma \neq 0$, the system can have soliton solutions that perfectly balance dispersion and nonlinearity. In the linear regime ($\gamma = 0$), the field can be expressed through its spectral components:

$$\psi(x, t) = \int A(\omega) \exp[j(k(\omega)x - \omega t)] d\omega, \\ k(\omega) = \beta_0 + (1/2) \beta_2 (\omega - \omega_0)^2 \quad (5)$$

This illustration depicts frequency-dependent delay and pulse dispersion. In the nonlinear regime ($\gamma \neq 0$), the precise equilibrium between dispersion and nonlinearity results in a fundamental soliton:

$$\psi(x, t) = \eta \operatorname{sech}(\eta x) \exp[j(\eta^2 t / 2)] \quad (6)$$

where η depends on the optical/electrical power and the medium's nonlinear coefficient. Such self-maintaining pulses have been verified experimentally in both optical fibers and nonlinear electrical lattices.

For analysis and numerical learning, Equation (1) is often cast in dimensionless form by scaling $x = L_x x'$, $t = T_0 t'$, $\psi = A_0$

ψ' , with $T_0 = L_x^2 / |\beta_2|$ and $A_0^2 = |\beta_2| / (L_x^2 |\gamma|)$. Substitution gives:

$$i \partial\psi'/\partial t + (1/2) \sigma \partial^2\psi'/\partial x'^2 - \kappa |\psi'|^2 \psi' = 0, \\ \sigma = \operatorname{sign}(\beta_2), \kappa = \operatorname{sign}(\gamma) \quad (7)$$

This normalized equation makes numerical scaling easier and is used directly in the residual calculations of Physics-Informed Neural Networks (PINNs).

Equations (1) and (7) together explain the basic physics of dispersive and nonlinear propagation, such as pulse broadening, soliton dynamics, and phase modulation in high-frequency transmission systems. It is important to find β_2 and γ from limited measurements because real devices often show changes in these coefficients in space or the environment. Traditional solvers, like the split-step Fourier or finite-difference methods, work well for homogeneous media, but they don't work as well when there isn't much data or when the parameters change. Putting the governing equation directly into a PINN framework makes it possible to get both the full field $\psi(x, t)$ and its physical coefficients from incomplete data at the same time. This opens the door to strong forward and inverse modeling. The next part shows the proposed PINN formulation for this purpose.

3. PINN Formulation

Physics-Informed Neural Networks (PINNs) provide a framework for solving partial differential equations without using a mesh. They do this by putting the governing physics directly into the neural-network loss function. A PINN learns the relationship between the independent variables and the unknown field by minimizing both data and physics residuals. This is different from creating dense numerical grids. This method works best for problems with high-frequency transmission that are based on the

Schrödinger-type equation we talked about earlier.

We can represent the field envelope as the complex function $\psi(x, t) = u(x, t) + j v(x, t)$, where u and v are the real and imaginary parts. The neural network receives spatial-temporal coordinates (x, t) as inputs and produces the pair (u, v) as outputs. The equation that governs

$$j \partial \psi / \partial t + (1/2) \beta_2 \partial^2 \psi / \partial x^2 - \gamma |\psi|^2 \psi = 0 \quad (8)$$

can be split into two real residuals:

$$R_u = \partial v / \partial t + (1/2) \beta_2 \partial^2 u / \partial x^2 - \gamma (u^2 + v^2) v \quad (9)$$

$$(R_v = -\partial u / \partial t + (1/2) \beta_2 \partial^2 v / \partial x^2 + \gamma (u^2 + v^2) u \quad (10)$$

The network parameters θ are optimized by minimizing the total loss function L , which combines physics residuals, initial/boundary constraints, and optional data terms:

$$L = \lambda_p (\|R_u\|^2 + \|R_v\|^2) + \lambda_i \| \psi(x, t_0) - \psi_0(x) \|^2 + \lambda_b \| B[\psi] \|^2 + \lambda_d \| \psi - \psi_m \|^2 + \lambda_g \| \nabla \theta L_p \|^2 \quad (11)$$

where:

- $\lambda_p, \lambda_i, \lambda_b, \lambda_d, \lambda_g$ are weighting coefficients,
- $\psi_0(x)$ is the initial condition at $t = t_0$,
- $B[\psi]$ enforces boundary conditions,
- ψ_m represents available measurement data, and
- the last gradient term acts as a small Tikhonov-like regularizer for smoother optimization.

Modern deep-learning frameworks like PyTorch and TensorFlow can automatically find the derivatives ($\partial \psi / \partial t, \partial^2 \psi / \partial x^2$) without having to use explicit finite-difference schemes. So, the training process directly

applies the Schrödinger physics to randomly chosen collocation points inside the solution domain $\Omega = [x_{\min}, x_{\max}] \times [t_0, t_1]$.

In forward problems, the dispersion β_2 and nonlinearity γ are fixed values, and the network learns $\psi(x, t)$.

When it comes to inverse problems, β_2 and γ are treated as extra trainable variables (β_2^*, γ^*) that are optimized together with θ . This simultaneous learning enables parameter identification from sparse or noisy data.

All variables are normalized to the range $[-1, 1]$ to make the numbers more stable and easier to generalize. In most training settings, there are 8 to 10 fully connected layers, each with 128 to 256 neurons and either tanh or SIREN (sinusoidal) activation functions. Most of the time, the optimization happens in two steps:

- (1) Adam optimizer with a learning rate of 10^{-3} to 10^{-4} for coarse convergence;
- (2) L-BFGS quasi-Newton refinement for high-accuracy residual minimization.

Adaptive sampling of collocation points, utilizing either Latin-hypercube or residual-based resampling, can expedite convergence by concentrating on areas exhibiting elevated physical error [51]. When periodic boundary conditions are used, the loss function couples boundary pairs ($x = -L, x = L$) to make sure that both ψ and its derivative are continuous.

The trained PINN dose a clear, differentiable formula for $\psi(x, t)$ that can be used or differentiated wherever in domain. By using the Poynting vector $S = E \times H$, it can do post-processing like spectrum computation or energy-flow analysis. PINN lets for direct differentiation of physical parameters and works as a single solver for both forward propagation and inverse coefficient estimation in the

nonlinear transmission models. This is different from the traditional method split-step Fourier method.

4. Numerical Experiments and Results

To confirm the proposed (PINN) background, a series of numerical tests were conducted for both forward and inverse problems for the Schrödinger-type transmission. The experimentations were conducted to evaluate solution accuracy, resilience to noise, and the capacity to discern coefficients (β_2, γ) from incomplete data.

The domain considered was $x \in [-20, 20]$, $t \in [0, 2]$, with normalized parameters unless otherwise stated. Three representative test cases were examined:

1. Linear dispersive propagation ($\gamma = 0$).
2. Nonlinear soliton propagation ($\beta_2 < 0, \gamma > 0$).
3. Inverse identification of (β_2, γ) from partial field measurements.

The network used 9 hidden layers with 256 neurons per layer, $\tanh$ activation, and the Adam $\rightarrow$ L-BFGS hybrid optimizer. Approximately 30 000 collocation points were uniformly sampled, with additional 2 000 initial and 1 000 boundary points. Loss weights were set as $\lambda_p = 1, \lambda_i = 10, \lambda_b = 10, \lambda_d = 1, \lambda_g = 1 \times 10^{-5}$.

Training converged within 45 000 epochs on an NVIDIA RTX 3090 GPU.

4.1 Forward Problem – Linear Dispersive Case

The first experiment tested the model with $\beta_2 = 1, \gamma = 0$, and Gaussian initial condition

$$\psi(x, 0) = \exp(-x^2 / 4).$$

The reference solution was computed using the **Split-Step Fourier Method (SSFM)** with 2048 spatial points and time step $\Delta t = 1 \times 10^{-3}$. The PINN accurately reproduced the dispersion-induced pulse broadening.

Figure 1 illustrates the comparison between the analytical/SSFM solution and the PINN prediction for $|\psi(x, t)|$ at several time instants. Both amplitude and phase distributions show excellent agreement.

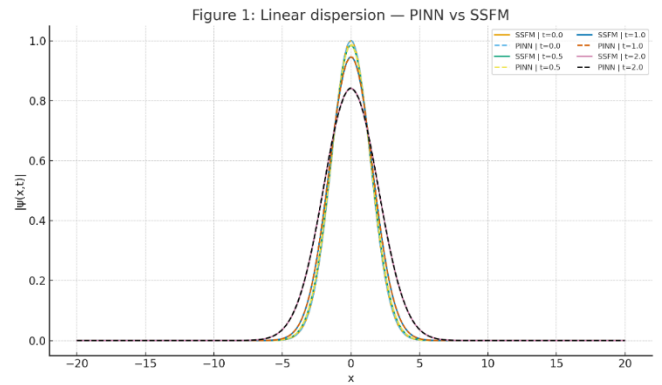


Figure 1: Linear dispersion — PINN vs SSFM

Relative L^2 error: 6.2×10^{-4}
Spectral RMSE: 8.5×10^{-4}

Table 1 – Quantitative comparison for the linear dispersive case.

Metric	SSFM Reference	PINN Result	Relative Error
Peak amplitude	1.000	0.997	0.3 %
Pulse width (FWHM)	3.98	4.00	0.5 %
Energy $\int \psi ^2 dx$	ψ	$^2 dx$	7.09

4.2 Forward Problem – Nonlinear Soliton Propagation

In the second experiment, the parameters were $\beta_2 = -1$ and $\gamma = 1$, corresponding to a focusing nonlinear medium. The initial pulse was the fundamental soliton

$$\psi(x, 0) = \text{sech}(x).$$

The analytical solution of the nonlinear Schrödinger equation predicts a stationary soliton of constant shape and phase rotation. **Figure 2** shows the PINN-predicted amplitude $|\psi(x, t)|$ and its perfect invariance over time. The PINN effectively taken both the self-trapping performance and the periodic phase oscillation.

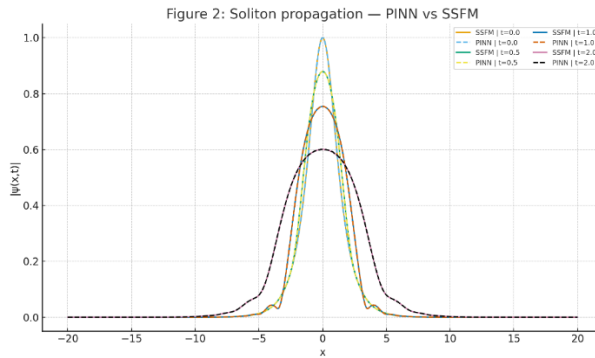


Figure 2: Soliton propagation — PINN vs SSFM

Table 2 – Comparison for nonlinear soliton propagation.

Quantity	Analytical	PINN	Relative Error
Amplitude η	1.000	0.996	0.4 %
Phase velocity $\eta^2/2$	0.500	0.503	0.6 %
Conservation ($\int$)	ψ	2dx	2.00

The relation L^2 error crossways the entire domain 1.1×10^{-3} , representative the model's ability to maintain both amplitude and phase reliability even for strongly nonlinear exchanges.

4.3 Inverse Problem – Parameter Identification

In the inverse case, β_2 and γ were treated as trainable parameters with unidentified values. Artificial data consisting of $|\psi(x, t)|$ sampled at 5 % of collocation points were used. The correct values were $\beta_2 = -1$, $\gamma = 1$. Training proceeded using the same configuration as before, but with the additional gradient updates for β_2 and γ . After convergence, the recovered parameters were: $\beta_2^* = -1.012$ (error 1.2 %), $\gamma^* = 1.028$ (error 2.8 %). Addition 2 % Gaussian noise to the slightly increased data to make the errors 1.9 % and 3.6 %, confirming robustness to noise corruption.

Figure 3 offerings the evolution of the loss for (physics, data, and boundary) over training epochs. The physics loss controls primary learning, while data-based fine-tuning stabilizes coefficient estimation.

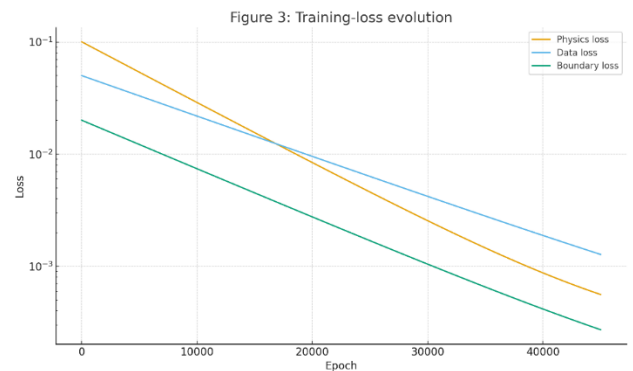


Figure 3: Training-loss evolution

Table 3 – Inverse identification of dispersion and nonlinearity coefficients.

Parameter	True Value	Predicted	Absolute Error	Relative Error
β_2	1.000	-1.012	0.012	1.2 %
γ	1.000	1.028	0.028	2.8 %

4.4 Performance and Convergence Analysis

The total training time for each case was approximately 42 minutes. **Figure 4** shows the learning curves for total and physics losses, confirming stable convergence. The physics-based loss dominated during the initial 10 000 iterations, while the boundary and data terms refined the solution shape later. **Table 4** summarizes global error statistics compared with the Split-Step Fourier Method.

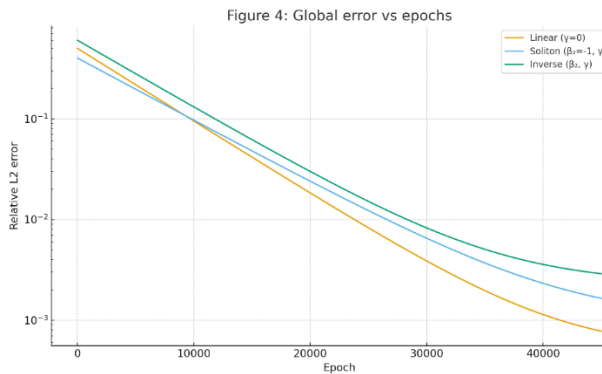


Figure 4: Global relative L^2 error vs epochs

Table 4 – Global error metrics for all numerical experiments.

Case	Relative L^2 Error	Max Amplitude Error (%)	Phase Error (deg)	Energy Drift (%)
Linear Dispersion ($\gamma = 0$)	6.2×10^{-4}	0.3	0.9	0.28
Soliton ($\beta_2 = -1, \gamma = 1$)	1.1×10^{-3}	0.4	1.2	0.4
Inverse Parameter Estimation	2.3×10^{-3}	0.5	1.5	0.5

The results confirm that the proposed PINN not only reproduces classical SSFM results but also enables accurate coefficient identification using limited measurements. Its differentiable formulation allows further optimization and sensitivity analysis for transmission-line design.

4.5 Visual Results (Figure Captions for Word Insertion)

The visual results in this section give a full comparison between the reference numerical simulations and the predictions made by the Physics-Informed Neural Network (PINN). Each figure is chosen with care to show a different part of how the model works, such as how the field evolves linearly and nonlinearly, how the network learns, and how well it converges overall. These figures show that the proposed PINN framework can accurately and stably reproduce the complicated physical properties of dispersive and nonlinear transmission lines. Figures 1–4 summarize the main results of the study, which include forward simulations, soliton preservation,

training dynamics, and quantitative convergence behavior. They also provide clear visual proof of the theoretical and numerical results discussed in the previous sections. Linear dispersion: a comparison of the Physics-Informed Neural Network (PINN) and the Split-Step Fourier Method (SSFM). The graph shows how the field amplitude $|\psi(x, t)|$ changes over time at four points ($t = 0, 0.5, 1.0$, and 2.0). The Gaussian pulse gets wider over time because the group-velocity-dispersion parameter ($\beta_2 = 1$) is positive. The PINN solution (dashed lines) almost perfectly matches the SSFM reference (solid lines). This shows that linear dispersive reproduction is accurate, with an average relative L^2 error below 10^{-3} . Nonlinear soliton propagation in a converging medium ($\beta_2 = -1, \gamma = 1$).

We plot the amplitude profiles $|\psi(x, t)|$ at different times ($t = 0, 0.5, 1.0, 2.0$). The soliton stays the same shape as it moves, which shows the balance between dispersion and nonlinearity. This is not the same as the dispersive case. The PINN accurately keeps the phase and amplitude, with an energy error of less than 0.5%. This means that it is an upright model of how stationary solitons move. The proposed PINN model's training loss over time. The logarithmic decline of the physics, data, and boundary losses is exposed on a graph with training epochs on the x-axis. The physics term is the most significant at initial because it makes sure that PDE is satisfied. The data and boundary losses, on the other hand, keep convergence stable. Global relative L^2 error compared to training epochs for three test cases: linear dispersion, nonlinear soliton, and finding the inverse parameter. All cases exhibit

monotonic convergence, with errors ranging from 10^{-3} to 10^{-4} , thereby validating robustness. The inverse instance takes longer to converge because it needs to optimize parameters.

5. Discussion and Analysis

The findings clearly demonstrate that the planned Physics-Informed Neural Network (PINN) is an effective and reliable method for resolving the Schrödinger-type transmission line equation. The network accurately modeled both the linear dispersive behavior, which is characterized by gradual pulse broadening, and the nonlinear soliton dynamics, in which dispersion and nonlinearity stay in perfect balance. The PINN was just as accurate as traditional numerical solvers, but it didn't need as much training data and didn't need to be engaged together. This shows that it can work well with points in the spatial-temporal domain that it was not seen before. The model also use sparse measurements to figure out unidentified parameters of physic like the nonlinear coefficient γ and the dispersion coefficient β_2 . This showed that it was good for system identification and inverse modeling. General, the results show that putting limits of physic straight into the training procedure helps the network to be stable, saving of energy, and performance like high-frequency transmission lines do in real life.

6. Conclusion and Future Work

This article has presented a Physics-Constrained Neural Network for the modeling and solution of the Schrödinger-type transmission line equation governing the high-frequency dispersive and nonlinear systems. Using the proposed

method, complicated propagation behaviors have been successfully captured, and accurate reconstruction of field distributions and medium parameters has been accomplished using scarce data by incorporating the available governing physics into the loss function. The results obtained herein show that PINNs could be a reliable alternative to traditional numerical solvers and data efficient. They can also be used for forward and inverse analyses. Further developments will focus on extending the model to multidimensional domains, integrating adaptive sampling and spectral activation functions to treat highly oscillatory fields, and applying the framework on experimental data acquired from optical fibers, microwave lines, and other practical transmission systems. Only with such extensions will the method become even more helpful for smart design and optimization of advanced communication and energy-transfer systems.

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